Robustness and spreading properties in real networks

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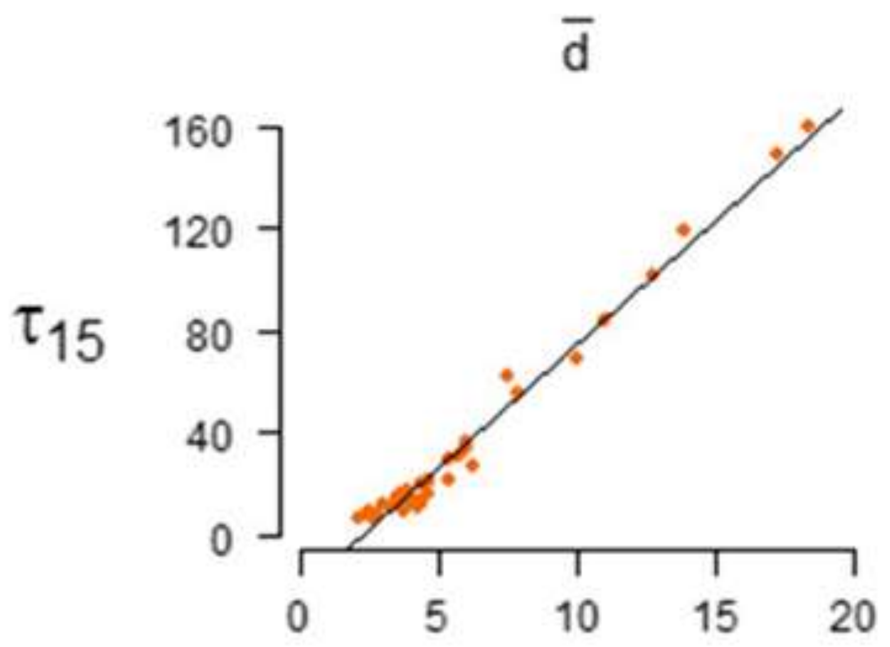
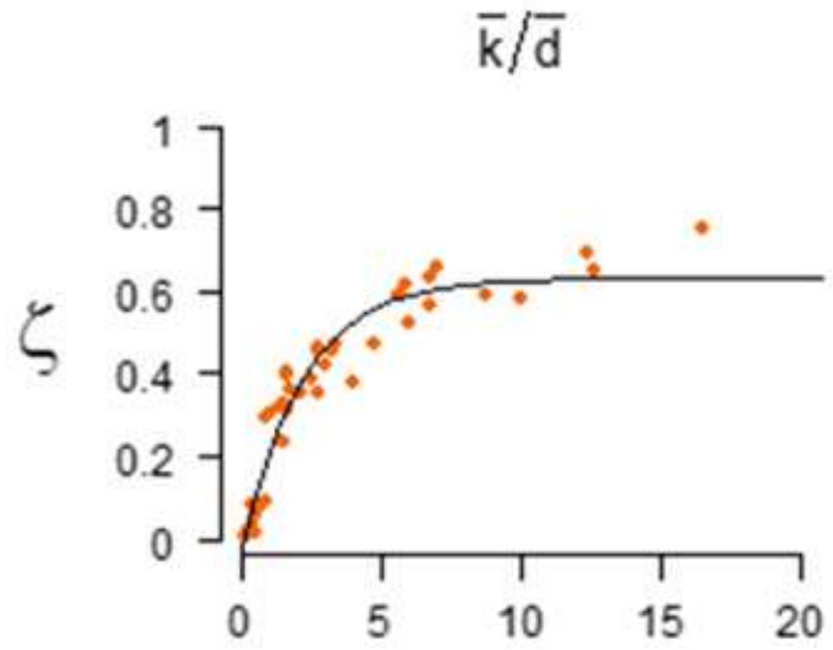
ID	Key	Full name	Formula	Definition	References
1	LD	Linkage density	$LD = \frac{L}{N}$	L is the link number and N is the number of nodes.	[22]
2	C	Connectance	$C = \frac{2L}{N \cdot (N-1)}$	L is the number of links and N is the number of nodes.	[22]
3	$\bar{k}$	Average node degree	$\bar{k} = \frac{1}{N} \cdot \sum_{i=1}^{i=N} k_i$	k_i is the degree of the node i, and N is the nodes number	[14]
4	μ_k	Node degree harmonic mean	$\mu_k = \frac{1}{N} \cdot \sum_{i=1}^{i=N} \frac{1}{k_i}$	k_i is the degree of the node i, and N is the number of network nodes.	New—from 3
5	σ_k^2	Node degree variance	$\sigma_k^2 = \frac{\sum_{i=1}^{i=N} (k_i - \bar{k})^2}{N-1}$	k_i is the degree of the node i, $\bar{k}$ the average node degree, and N is the nodes number.	New—from 3
6	σ_k	Node degree standard deviation	$\sigma_k = \sqrt{\frac{\sum_{i=1}^{i=N} (k_i - \bar{k})^2}{N-1}}$	k_i is the degree of the node i, $\bar{k}$ the average node degree, and N is the nodes number.	New—from 3
7	$n\sigma_k$	Node degree normalized standard deviation	$n\sigma_k = \frac{\sigma_k}{\bar{k}}$	σ_k is the standard deviation of the node degree and $\bar{k}$ the average node degree.	New—from 3
8	$K1$	Degree 1	$K1 = \frac{N_{(k=1)}}{N}$	$N_{(k=1)}$ is the number of nodes of degree $k = 1$, and N is the nodes number.	New—from 3
9	$K2$	Degree 2	$K2 = \frac{N_{(k \le 2)}}{N}$	$N_{(k \le 2)}$ is the number of nodes of degree $k \le 2$, and N is the nodes number.	New—from 3
10	Hub	Hub index	$Hub = \frac{K_{max}^{1\%}}{K}$	where $K_{max}^{1\%}$ is the sum of the degree of the 1% of the most connected nodes, and K is sum of the degree of all nodes in the networks.	New—from 3
11	AH	Albertson index	$AH = \sum_{i,j \in L} \lvert k_i - k_j \rvert$	i,j is the link connecting nodes i and j, k_i is the degree of the node i, k_j the degree of the node j, and L is the network links set.	[23]
12	nAH	Normalized Albertson index	$nAH = \frac{AH}{L}$	AH is the Albertson index and L the number of links.	New—from 12
13	EH	Estrada index	$EH = \sum_{i,j \in L} (k_i^{-1/2} - k_j^{-1/2})^2$	i,j is the link connecting nodes i and j, k_i is the degree of the node i and k_j is the degree of the node j, and L is the network links set.	[24]
14	H	Node degree Shannon index	$H = \frac{-\sum_{i=1}^{N} k_i \cdot \log(k_i)}{\log(N)}$	k_i is the degree of the nodes i and N is the number of nodes.	[25]
15	A	Network assortativity	$r = \frac{1}{\sigma_q^2} \sum_{j,k \in N} jk(e_{jk} - q_j q_k)$	σ_q is the standard deviation of the excess degree distribution, e_{jk} is the fraction of links connecting nodes of degree j and k, and q_j, q_k are the excess degree of nodes of degree j and k.	[18]
16	$\bar{d}$	Average node distance	$\bar{d} = \frac{1}{N \cdot (N-1)} \sum_{i,j \in N, i \ne j} d_{ij}$	d_{ij} is the distance between nodes i and j and N the nodes number.	[14]
17	μ_d	Node distance harmonic mean	$\mu_d = \frac{\sum_{i,j=1}^{i,j=N} \frac{1}{d_{ij}}}{N}$	d_{ij} is the distance between i and j and N nodes number	New—from 16
18	σ_d	Node distance standard deviation	$\sigma_d = \sqrt{\frac{\sum_{i,j=1 (i \ne j)}^{i,j=N} (d_{ij} - \bar{d})^2}{N \cdot (N-1)}}$	d_{ij} is the distance between node i and node j, $\bar{d}$ is the average node distance and N is nodes number	New—from 16
19	$n\sigma_d$	Node distance normalized standard deviation	$n\sigma_d = \frac{\sigma_d}{\bar{d}}$	σ_d is the node distance standard deviation, $\bar{d}$ is the average node distance and N is the nodes number.	New—from 18
20	W	Wiener index	$W = \sum_{i,j \in N, i \ne j} d_{ij}$	d_{ij} is the distance between node i and node j, $\bar{d}$ is the average node distance and N is the nodes number.	[26]
21	Φ	Network eccentricity	$\Phi = \frac{1}{N} \sum_{i=1}^{i=N} \varepsilon(i)$	$\varepsilon(i)$ is the eccentricity of the node i and N is the nodes number.	[14]
22	$n\Phi$	Normalized network eccentricity	$n\Phi = \frac{\Phi}{\bar{d}}$	Φ is the average network eccentricity and $\bar{d}$ the average node distance	New—from 21
23	D	Network diameter	$D = \max_{i,j \in N, i \ne j} (d_{ij})$	d_{ij} is the distance between i and j and N the nodes number	[14]
24	nD	Normalized diameter	$nD = \frac{D}{\bar{d}}$	D is the network diameter and $\bar{d}$ the average node distance.	New—from 23
25	π	Network radius	$\pi = \min_{i \in G} (\varepsilon(i))$	$\varepsilon(i)$ is the eccentricity of the node i.	[14]
26	$n\pi$	Normalized network radius	$n\pi = \frac{\pi}{\bar{d}}$	π is the network diameter and $\bar{d}$ the average node distance.	New—from 25
27	π/D	Radius-diameter ratio	$\frac{\pi}{D} = \frac{\min (\varepsilon(i))}{\max (\varepsilon(i))}$	$\varepsilon(i)$ is the eccentricity of the node i.	New—from 23 to 25
28	$n(\pi/D)$	Radius-diameter normalized ratio	$n(\pi/D) = \frac{\min (\varepsilon(i))}{\max (\varepsilon(i))} \cdot \frac{1}{\bar{d}}$	$\varepsilon(i)$ is the eccentricity of the node i and $\bar{d}$ the average node distance.	New—from 27
29	Eff	Network efficiency	$Eff = \frac{\sum_{i \ne j \in G} \frac{1}{d_{ij}}}{N \cdot (N-1)}$	d_{ij} is the distance between node i and node j and N the nodes number	[27]
30	Com	Network communicability	$Com = \frac{1}{N \cdot (N-1)} \cdot \sum_{p,q \in N, p \ne q} G_{pq}$	N is the number of network nodes, and G_{pq} is the communicability between node p and q.	[28]
31	$\ln Com$	Network communicability logarithm	$\ln Com = \log_e (Com)$	Com is the communicability of the network	New—from 30
32	T	Average node transitivity	$T = \frac{1}{N} \cdot \sum_{i=1}^{i=N} \tau_i$	τ_i is the transitivity of the node i, and N is the nodes number	[13]
33	B	Average node betwenness	$B = \frac{1}{N} \cdot \sum_{i=1}^{i=N} g(i)$	N is the number of nodes and $g(i)$ the betwenness of the node i.	[29]
34	nB	Average normalized node betwenness	$nB = \frac{1}{N} \cdot \sum_{i=1}^{i=N} {}^{n}g(i)$	N is the number of nodes, ${}^{n}g(i)$ is the normalized betwenness of the node i.	[29]
35	Clo	Average node closeness	$Clos = \frac{1}{N} \sum_{i=1}^{N} C_i$	C_i is the closeness of the node i and N is the nodes number.	[29]
36	$nClo$	Average normalized node closeness	$nClos = \frac{1}{N} \sum_{i=1}^{N} nC_i$	nC_i is the normalized closeness of the node i and N is the nodes number.	New—from 35
37	$\overline{k_s}$	Average node coreness	$\overline{k_s} = \frac{1}{N} \sum_{i=1}^{N} k_i^s$	k_i^s is the coreness of the node i and N is the number of nodes.	[5]
38	Q	Network modularity	$Q = \frac{1}{2L} \sum_{i,j} (a_{ij} - \frac{k_i k_j}{2L}) \delta(c_i, c_j)$	L is the number of links, a_{ij} is the element of the A adjacency matrix in row i and column j, i, k_i is the degree of i, k_j is the degree of j, c_i is the module (or community) of node i, c_j that of j, the sum goes over all i and j pairs of nodes, and $\delta(x, y)$ is 1 if x = y and 0 otherwise.	[30]
39	$\bar{k}/\bar{d}$	$\bar{k}/\bar{d}$ index	$\bar{k}/\bar{d} = \frac{\frac{1}{N} \sum_{i=1}^{i=N} k_i}{\frac{1}{N \cdot (N-1)} \sum_{i,j \in N, i \ne j} d_{ij}}$	$\bar{k}$ is the average node degree, $\bar{d}$ is the average node distance.	[15]
40	BB	Centricity index	$BB = \sum_{i=1}^{N} \frac{k_i}{v_i}$	k_i is the degree of node i, v_i is the farness of node i.	[15]

Predicting real networks SIR spreading with 40 network structural indicators



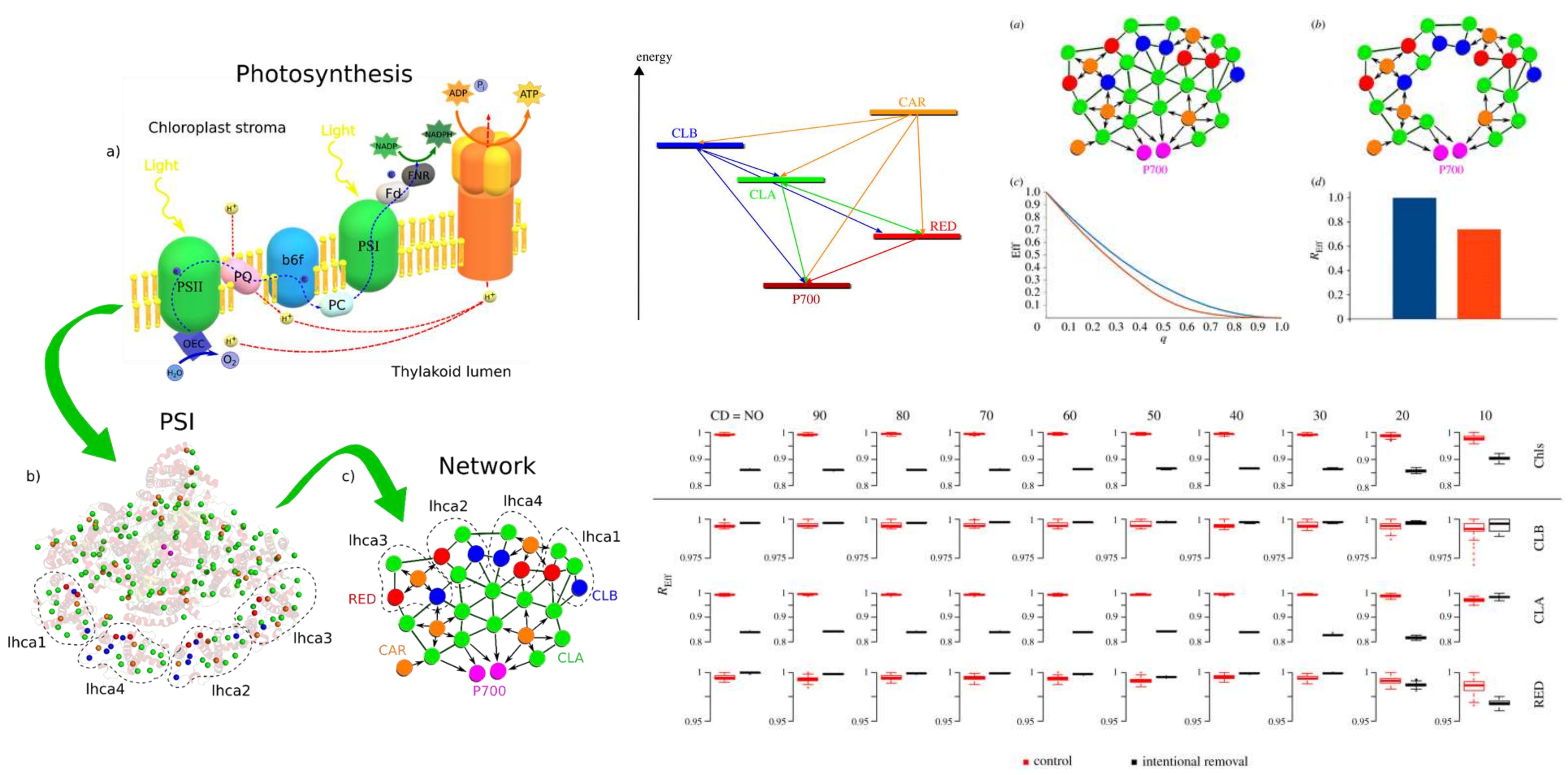
Notions based on the node distance are best predictors of the SIR spreading pace



“Topological complexity Index” are best predictor of the epidemic peak

Bellingeri M., D Bevacqua, M Turchetto, F Scotognella, R Alfieri, et al. 2022., Network structure indexes to forecast epidemic spreading in real-world complex networks, Frontiers in Physics, 10, 1121.

Photosystem I the *P. sativum* as a complex network

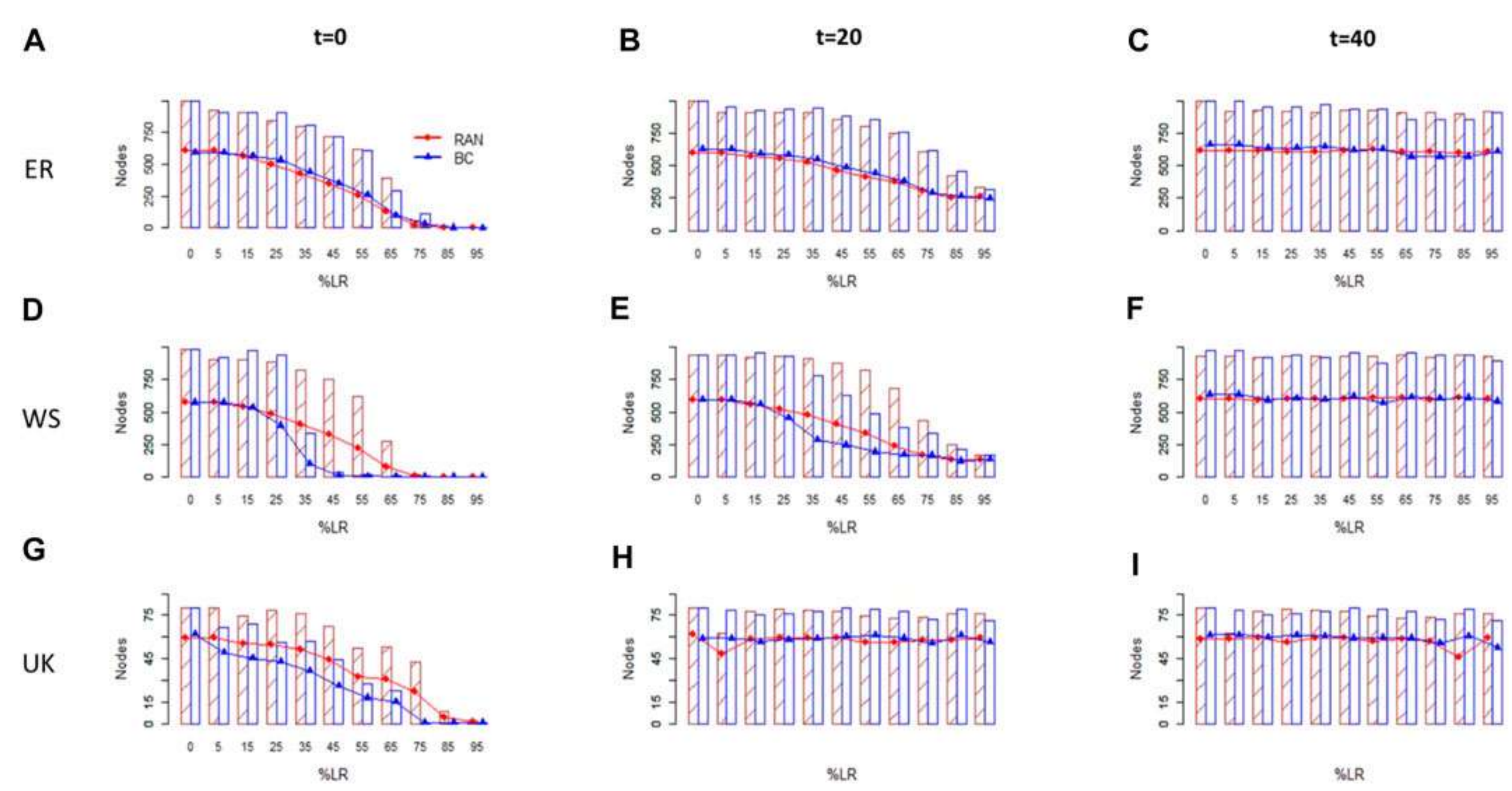


CLA removal triggers the fastest efficiency decrease

CLAs are the main contributors EET efficiency

Montepietra D., Bellingeri M., Ross A. M., Scotognella F. and Cassi D. 2020. Modelling photosystem I as a complex interacting network. J. R. Soc. Interface 17 20200813.

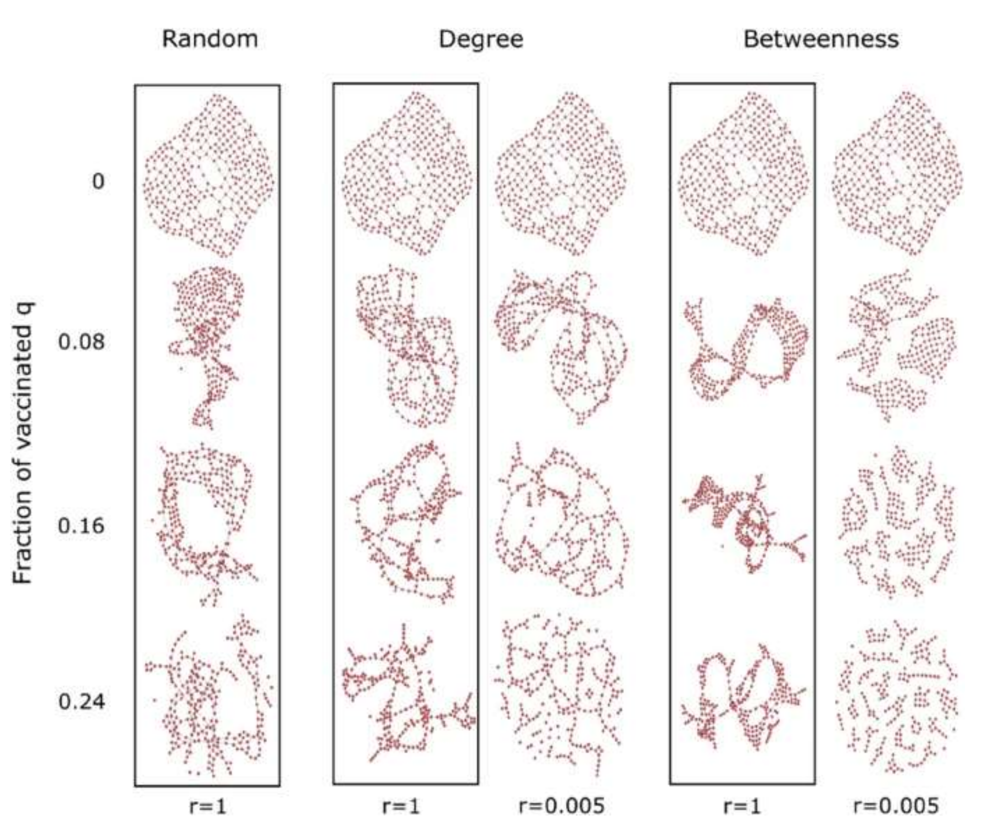
Model Social Network Distancing

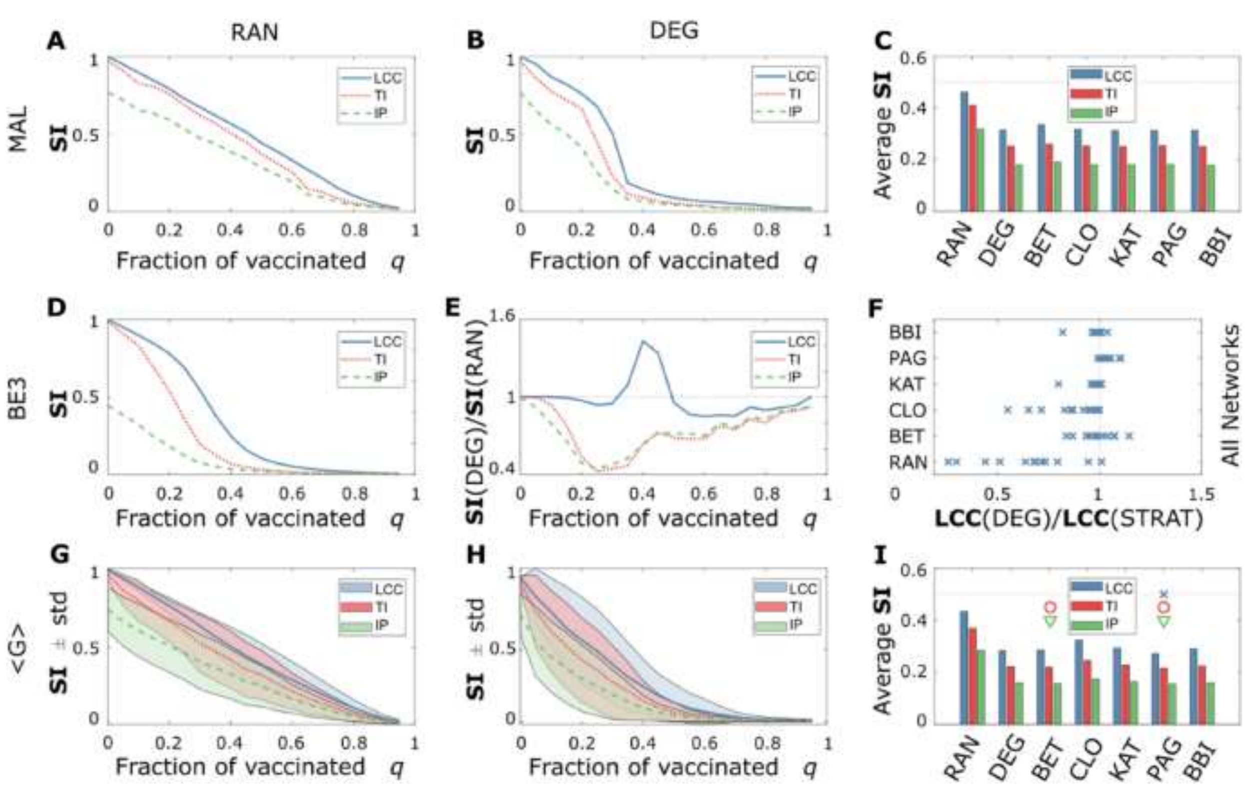


Peak (*Lines*) and total (*Bars*) infected as a function of the fraction of the links removed (%LR).

Bellingeri, M., Turchetto, M., Bevacqua, D., Scotognella, F., Alfieri, R., Nguyen, Q., & Cassi, D. (2021). Modeling the Consequences of Social Distancing Over Epidemics Spreading in Complex Social Networks: From Link Removal Analysis to SARS-CoV-2 Prevention. Frontiers in Physics, 9.

Vaccination strategies as node removal on real-world networks





- Largest connected component (LCC)
- Total number of infected (TI)
- Infected peak (IP)

Best strategy depends on available vaccines

Partial recalculation of the node centrality increases efficacy by up to 80%.

Sartori, F., Turchetto, M., Bellingeri, M., Scotognella, F., Alfieri, R., Nguyen, N., Le, T., Nguyen, Q., & Cassi, D. (2022). A comparison of node vaccination strategies to halt SIR epidemic spreading in real-world complex networks. Scientific Reports, 12(1), 1-13.

PGR Italia-Vietnam. Information and Communication Technologies: “*Robustness and important components of real-world social weighted networks*”

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